

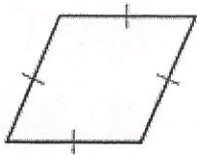
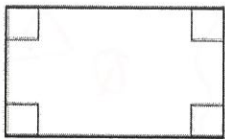
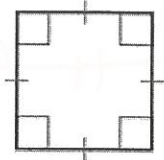
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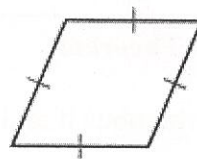
Properties of Special Parallelograms

on Objective INBAT use properties of special ||-grams;
use coordinate geometry to identify special ||-grams

DEFINITIONS		
Rhombus	Rectangle	Square
		
a -gram w/ 4 $\cong$ sides	a -gram w/ 4 right $\angle$ s	a -gram w/ 4 $\cong$ sides AND 4 right $\angle$ s

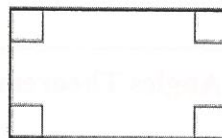
Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.



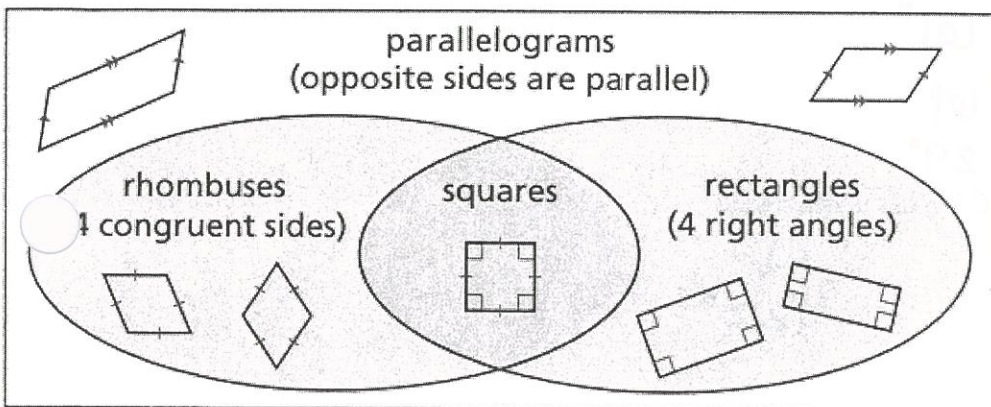
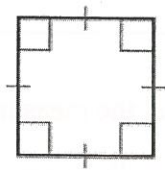
Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.



Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.



*rect/sq/rhombus
have all properties
of ||-grams!

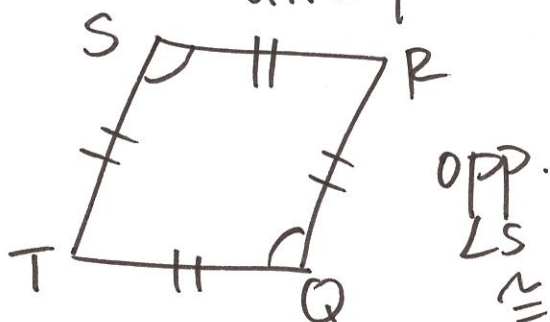
- ① opp. sides $\parallel$
- ② opp. sides $\cong$
- ③ opp. $\angle$ s $\cong$
- ④ diags. bisect each other

Example

1. For any rhombus QRST, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a diagram. Remember, the order of the vertices in the name "QRST" tells you the order of how to "connect" the vertices!

a. $\angle Q \cong \angle S$

always!



b. $\angle Q \cong \angle R$

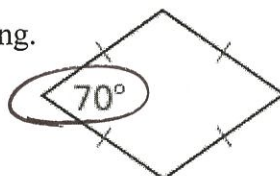
Sometimes!



2. Classify the special quadrilateral. Explain your reasoning.

rhombus!

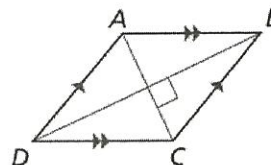
4 $\cong$ sides



(not a rectangle b/c no rt. $\angle$ s)
(not a square b/c not a rect.)

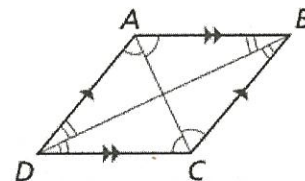
Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.



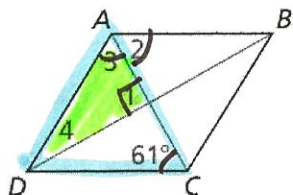
Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.



Example

3. Find the measures of the numbered angles in rhombus ABCD.



$$\begin{aligned} m\angle 1 &= 90^\circ \\ m\angle 2 &= 61^\circ \\ m\angle 3 &= 61^\circ \\ m\angle 4 &= 29^\circ \end{aligned}$$

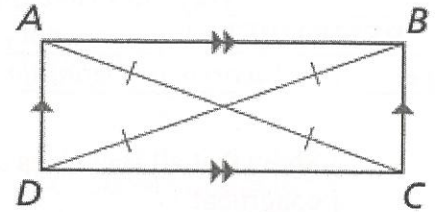
4. In the diagram above, what is $m\angle ADC$? What is $m\angle BCD$?

$\downarrow$
 58°

$\downarrow$
 122°

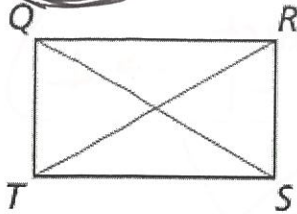
Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.



Example

5. In rectangle QRST, $QS = 5x - 31$ and $RT = 2x + 11$. Find the lengths of the diagonals of QRST.



$$5x - 31 = 2x + 11$$

$$x = 14$$

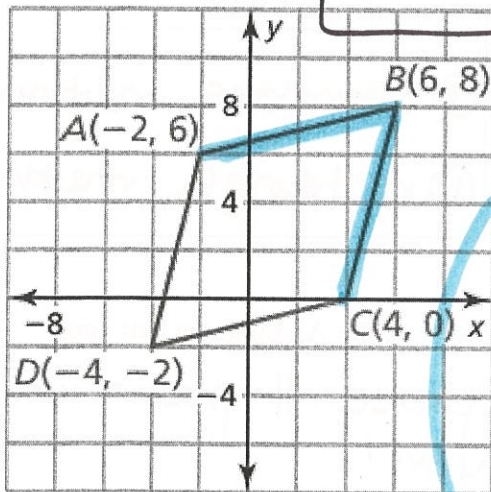
$$QS = RT = 39$$

Think About It... What should we show to prove that a parallelogram is a square? Hint: Use the Square Corollary...

show it is a rhombus, then show it is a rectangle. (or vice-versa)

Example

6. Decide whether quadrilateral ABCD below is a rectangle, rhombus, or square. Give ALL names that apply.



first show it is a ||-gram
slope $\overline{AB} = \frac{1}{4}$ slope $\overline{CD} = \frac{1}{4}$

slope $\overline{BC} = 4$ slope $\overline{AD} = 4$

Since both pairs of opp. sides in the quad. have the same slope, then both pairs of opp. sides are ||. So, ABCD is a ||-gram.

rhombus?

$$\text{slope } \overline{AC} = \frac{-3}{3} = -1$$

$$\text{slope } \overline{BD} = \frac{5}{5} = 1$$

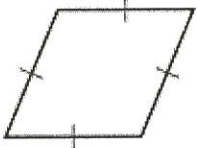
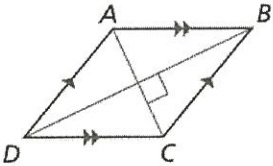
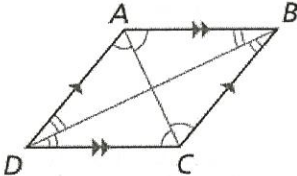
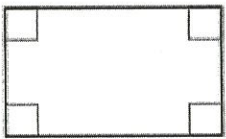
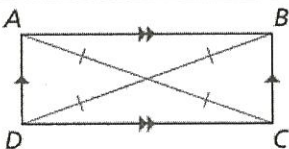
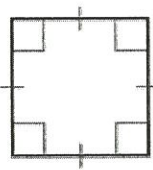
Since the product of the slopes of the diags of the ||-gram = -1 , the diags are $\perp$. So, ABCD is

rectangle?

Since the product of slopes of consec. sides of ||-gram $\neq -1$, then $\angle B$ is not a right $\angle$. So, ABCD is not a

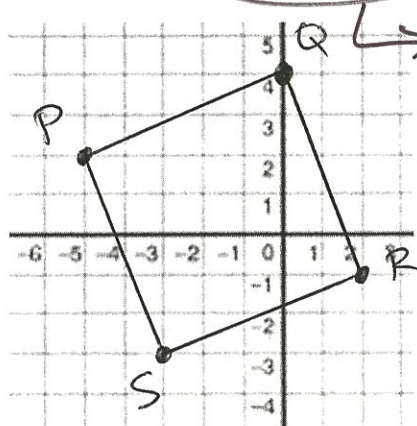
square?

ABCD is not a square because it is not a rectangle.

Ways to Prove that a Quadrilateral is a SPECIAL Parallelogram			
REMEMBER: If you are not given it is a $\parallel$ -gram, then you must first prove that it is a $\parallel$ -gram!			
Shape	Theorem or Definition	Diagram	Method for Coordinate Plane
Rhombus	Show that all four sides are congruent		dist. formula for all 4 sides (if $= \rightarrow \cong$)
	Show that diagonals are perpendicular		slopes of each diag (if product $= -1 \rightarrow \perp$)
	Show that diagonals bisect the angles		
Rectangle	Show that all four angles are right angles		slopes of each side (if product of consec. sides $= -1 \rightarrow \perp \rightarrow \text{rt. } \angle$)
	Show that diagonals are congruent		dist. formula for each diag (if $= \rightarrow \cong$)
Square	Show it is a rhombus AND a rectangle!		① rhombus method ① rectangle method

Example

7. Decide whether parallelogram PQRS is a rectangle, rhombus, or square. Give ALL names that apply.



→ don't need to prove $\parallel$ -gram i/c it was given!